Example 10-10: Consider the example

$$y[n] = 0.8y[n-1] + 2x[n] + 2x[n-1]$$

In order to define the filter coefficients in MATLAB, we put all the terms with $y[n]$ on one side of the equation, and the terms with $x[n]$ on the other.

$$y[n] - 0.8y[n-1] = 2x[n] + 2x[n-1]$$

Then we read off the filter coefficients and define the vectors `aa` and `bb` as

```
aa = [ 1, -0.8 ]        bb = [ 2, 2 ]
```

Thus, the vectors `aa` and `bb` are in the same form as for the `filter` function. The following call to `freqz` generates a 401-point vector `HH` containing the values of the frequency response at the vector of frequencies specified by the third argument, `2*pi*[-1:.005:1]`:

```
HH = freqz( bb, aa, 2*pi*[-1:.005:1] );
```

Plots of the resulting magnitude and phase are shown in Fig. 10-8. An extended frequency interval $-2\pi \le \hat{\omega} \le +2\pi$ is shown so that the 2π-periodicity of $H(e^{j\hat{\omega}})$ is evident.[1]

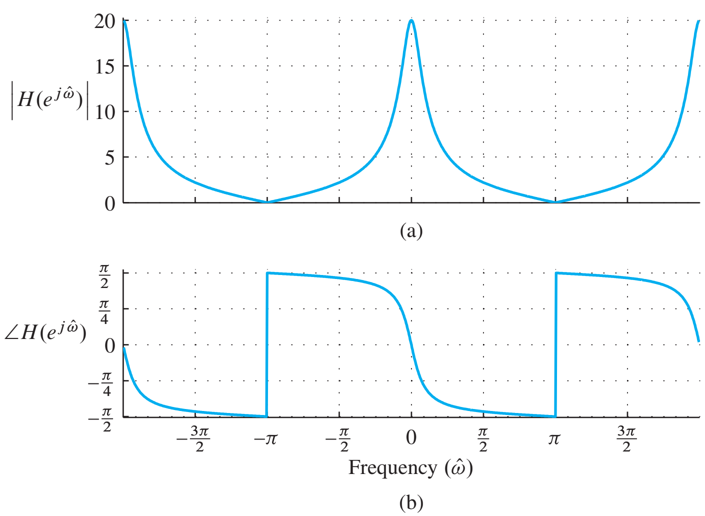

Figure 10-8: Frequency response (magnitude and phase) for a first-order feedback filter. The pole is at $z = 0.8$, and the numerator has a zero at $z = -1$.

[1]The labels on the graph were created using special plotting functions.

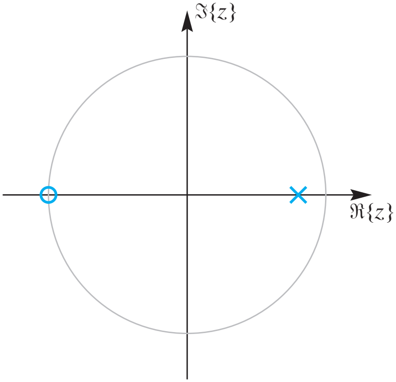

Figure 10-9: Pole-zero plot for a first-order IIR filter. The pole is at $z = 0.8$, and the numerator has a zero at $z = -1$.

In this example, we can look for a connection between the poles and zeros and the shape of the frequency response. From the coefficients of the difference equation, we obtain the system function

$$H(z) = \frac{2 + 2z^{-1}}{1 - 0.8z^{-1}}$$

which, as shown in Fig. 10-9, has a pole at $z = 0.8$ and a zero at $z = -1$.

The point $z = -1$ is the same as $\hat{\omega} = \pi$ because $z = -1 = e^{j\pi} = e^{j\hat{\omega}}\big|_{\hat{\omega}=\pi}$. Thus, $H(e^{j\hat{\omega}})$ has the value zero at $\hat{\omega} = \pi$, since $H(z)$ is zero at $z = -1$. In a similar manner, the pole at $z = 0.8$ has an effect on the frequency response near $\hat{\omega} = 0$. Since $H(z)$ blows up at $z = 0.8$, the nearby points on the unit circle must have large values. The closest point on the unit circle is at $z = e^{j0} = 1$. In this case, we can evaluate the frequency response directly from the $H(z)$ formula to get

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=0} = H(z)\Big|_{z=1} = \frac{2 + 2z^{-1}}{1 - 0.8z^{-1}}\Bigg|_{z=1} = \frac{2 + 2(1)}{1 - 0.8(1)} = \frac{4}{0.2} = 20$$

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.