Example 10-12: In this case, define $Y(z)$ with a second-order numerator as

$$Y(z) = \frac{2 - 2.4z^{-1} - 0.4z^{-2}}{1 - 0.3z^{-1} - 0.4z^{-2}} = \frac{2 - 2.4z^{-1} - 0.4z^{-2}}{(1 + 0.5z^{-1})(1 - 0.8z^{-1})}$$

Now we must add a constant term to the partial fraction expansion, otherwise, we cannot generate the term $-0.4z^{-2}$ in the numerator when we combine the partial fractions over a common denominator—that is, we must assume the following form for $Y(z)$:

$$Y(z) = \frac{A}{1 + 0.5z^{-1}} + \frac{B}{1 - 0.8z^{-1}} + C$$

How can we determine the constant C? One way is to perform long division of the denominator polynomial into the numerator polynomial until we get a remainder whose degree is lower than that of the denominator. In this case, the polynomial long division is carried out as follows:

$$\begin{array}{r|l} & \quad 1 \\ \cline{2-2} -0.4z^{-2} - 0.3z^{-1} + 1 & -0.4z^{-2} - 2.4z^{-1} + 2 \\ & \underline{-0.4z^{-2} - 0.3z^{-1} + 1} \\ & \qquad \underbrace{-2.1z^{-1} + 1}_{\text{remainder}} \end{array}$$

Thus, if we place the remainder $(1 - 2.1z^{-1})$ over the denominator (in factored form), we can write $Y(z)$ as a rational part (fraction) plus the constant 1, that is,

$$Y(z) = \frac{1 - 2.1z^{-1}}{(1 + 0.5z^{-1})(1 - 0.8z^{-1})} + 1$$

The next step is to apply the partial fraction expansion technique to the rational part of $Y(z)$. Since the rational part turns out to be identical to $X(z)$ in (10.8) from Example 10-11, the results would be the same as in that example, so we can write $Y(z)$ as

$$Y(z) = \frac{2}{1 + 0.5z^{-1}} - \frac{1}{1 - 0.8z^{-1}} + 1$$

Therefore, from entries 4 and 6 in Table **??**,

$$y[n] = 2(-0.5)^n u[n] - (0.8)^n u[n] + \delta[n]$$

Notice again that the time-domain sequence has terms of the form $p_k^n u[n]$. The constant term in the system function is the z-transform of an impulse, which is nonzero only at $n = 0$ (entry 4 in Table **??**).

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