Example 10-13: If $b_0 = 5$, $a_1 = -0.8$, and $\hat{\omega}_0 = 2\pi/10$, the transient component in (**??**) is

$$\begin{aligned} y_t[n] &= \left(\frac{-4}{-0.8 - e^{j0.2\pi}}\right)(-0.8)^n u[n] \\ &= 2.3351 e^{-j0.3502}(-0.8)^n u[n] \\ &= 2.1933(-0.8)^n u[n] - j0.8012(-0.8)^n u[n] \end{aligned}$$

Similarly, the steady-state component in (**??**) is

$$\begin{aligned} y_{ss}[n] &= \left(\frac{5}{1 + 0.8e^{-j0.2\pi}}\right) e^{j0.2\pi n} u[n] \\ &= 2.9188 e^{j0.2781} e^{j0.2\pi n} u[n] \\ &= 2.9188 \cos(0.2\pi n + 0.2781)\, u[n] + j\, 2.9188 \sin(0.2\pi n + 0.2781)\, u[n] \end{aligned}$$

Figure **??** shows the real parts of the total output (a), the transient component (b), and the steady-state component (c). The signals all start at $n = 0$ which is when the complex exponential is applied at the input. Note how the transient component $(-0.8)^n$ oscillates, but dies away, which explains why the steady-state component eventually equals the total output. In Fig. **??**, $y[n]$ in (a) and $y_{ss}[n]$ in (c) look identical for $n > 15$.

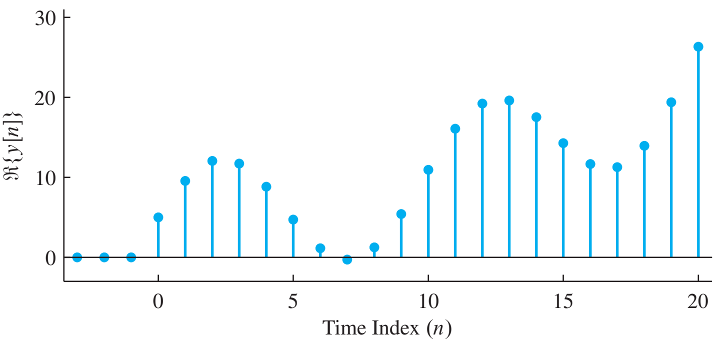

Figure 10-15: Illustration of an unstable IIR system where the output is dominated by the growing exponential component. Pole is at $z = 1.1$.

On the other hand, if the pole were at $z = 1.1$, the system would be unstable and the output would "blow up" as shown in Fig. 10-15. In this case, the output contains a term $(1.1)^n$ that eventually dominates and grows without bound.

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