Example 10-15: Assume that the system function is

$$H(z) = \frac{1}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})} \tag{10.10}$$

After factoring the denominator, we see that the poles are at $z = \frac{1}{2}$ and $z = \frac{1}{3}$. In addition, there are two zeros at $z = 0$. The poles and zeros of $H(z)$ are plotted in Fig. **??**. We can extract the filter coefficients directly from the denominator of $H(z)$ and write the following difference equation

$$y[n] = \tfrac{5}{6}y[n-1] - \tfrac{1}{6}y[n-2] + x[n] \tag{10.11}$$

which must be satisfied for any input and its corresponding output. Specifically, the impulse response would satisfy the difference equation

$$h[n] = \tfrac{5}{6}h[n-1] - \tfrac{1}{6}h[n-2] + \delta[n] \tag{10.12}$$

which can be iterated to compute $h[n]$ if we know the values of $h[-1]$ and $h[-2]$. Specifically, to start the iteration, we need to know the values of the impulse response sequence just prior to $n = 0$ where the impulse first becomes nonzero. These values are supplied by the initial rest condition, which means that $h[-1] = 0$ and $h[-2] = 0$. The following table shows the computation of a few values of the impulse response:

n	$n < 0$	0	1	2	3	4	$\cdots$
$x[n]$	0	1	0	0	0	0	$\cdots$
$h[n-2]$	0	0	0	1	$\frac{5}{6}$	$\frac{19}{36}$	$\cdots$
$h[n-1]$	0	0	1	$\frac{5}{6}$	$\frac{19}{36}$	$\frac{65}{216}$	$\cdots$
$h[n]$	0	1	$\frac{5}{6}$	$\frac{19}{36}$	$\frac{65}{216}$	$\frac{211}{1296}$	$\cdots$

In contrast to the simpler first-order case, iteration is not too helpful because it is very difficult to guess the general n^{th} term for the impulse response sequence. Fortunately, we can rely on the inverse z-transform technique to give us the general formula. Applying the partial fraction expansion to (10.10), we obtain

$$H(z) = \frac{3}{1 - \frac{1}{2}z^{-1}} - \frac{2}{1 - \frac{1}{3}z^{-1}}$$

Then the inverse z-transform yields $h[n] = 3(\frac{1}{2})^n\, u[n] - 2(\frac{1}{3})^n\, u[n]$. Since both poles are inside the unit circle, the impulse response dies out for n large (i.e., the system is stable).

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