Example 10-17: As an example of an oscillator with a different frequency, we can use (**??**) to define a difference equation with prescribed pole locations. If we take $r = 1$ and $\theta = \pi/2$, as shown in Fig. 10-21, we get $a_1 = 2r\cos\theta = 0$ and $a_2 = -r^2 = -1$, and the difference equation (**??**) becomes

$$y[n] = -y[n-2] + x[n] \tag{10.14}$$

The system function of this system is

$$\begin{aligned} H(z) = \frac{1}{1+z^{-2}} &= \frac{1}{(1-e^{j\pi/2}z^{-1})(1-e^{-j\pi/2}z^{-1})} \\ &= \frac{\frac{1}{2}}{1-e^{j\pi/2}z^{-1}} + \frac{\frac{1}{2}}{1-e^{-j\pi/2}z^{-1}} \end{aligned}$$

The inverse z-transform gives a general formula for $h[n]$:

$$h[n] = \tfrac{1}{2}e^{j(\pi/2)n}u[n] + \tfrac{1}{2}e^{-j(\pi/2)n}u[n] = \cos\left((2\pi/4)n\right)u[n] \tag{10.15}$$

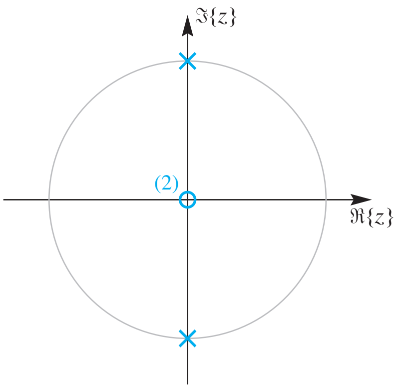

Figure 10-21 Pole-zero plot for system with $H(z) = \dfrac{1}{1+z^{-2}}$. The poles are on the unit circle at $z = \pm j$, and there are two zeros at $z = 0$.

Once again, the frequency of the cosine term in the impulse response is equal to the angle of the pole, $\pi/2 = (2\pi/4)$.

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.