Example 10-18: As an example of a stable system, we make the radius of the poles less than one with $r = \frac{1}{2}$ and the angles $\pm\pi/3$, as shown in Fig. **??**. Then the filter coefficients are $a_1 = 2r\cos\theta = 2(\frac{1}{2})(\frac{1}{2}) = \frac{1}{2}$ and $a_2 = -r^2 = -(\frac{1}{2})^2 = -\frac{1}{4}$, so the difference equation (**??**) becomes

$$y[n] = \tfrac{1}{2}y[n-1] - \tfrac{1}{4}y[n-2] + x[n] \tag{10.16}$$

The system function of this system is

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}} = \frac{\frac{1}{\sqrt{3}}e^{-j\pi/6}}{1 - \frac{1}{2}e^{j\pi/3}z^{-1}} + \frac{\frac{1}{\sqrt{3}}e^{j\pi/6}}{1 - \frac{1}{2}e^{-j\pi/3}z^{-1}}$$

and the inverse z-transform of the right-hand side gives the general formula for $h[n]$ after combining two complex exponential terms into a real sinusoid as

$$h[n] = \tfrac{2}{\sqrt{3}}(\tfrac{1}{2})^n \cos\left((2\pi/6)n - \pi/6\right) u[n] \tag{10.17}$$

In this case, the general formula for $h[n]$ has an exponential decay of $(\frac{1}{2})^n$ multiplying a periodic cosine with period 6. The frequency of the cosine term in the impulse response (10.17) is again the angle of the pole, $\pi/3 = 2\pi/6$; while the decaying term $r^n = (\frac{1}{2})^n$ is controlled by the radius of the pole r. Figure 10-23 shows $h[n]$ for this example. Note the rapid decay of the impulse response. This rapid decay can be predicted from the pole-zero plot in Fig. **??**, since, in general, poles that are not close to the unit circle correspond to rapidly decaying exponential components of the impulse response.

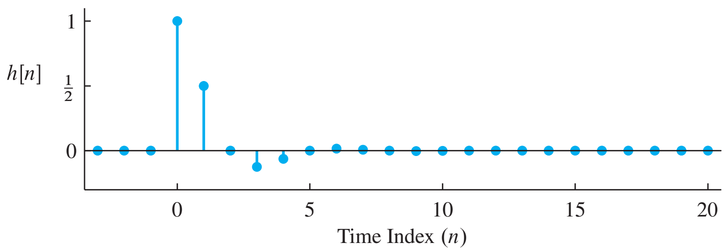

Figure 10-23: Impulse response for a two-pole system with system function $H(z) = \frac{1}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$.

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