Example 10-20: Consider the system introduced in Example 10-19:

$$y[n] = 0.9y[n-1] - 0.81y[n-2] + x[n] - x[n-2] \tag{10.19}$$

In order to define the filter coefficients in MATLAB, we put all the terms with $y[n]$ on one side of the equation, and the terms with $x[n]$ on the other.

$$y[n] - 0.9y[n-1] + 0.81y[n-2] = x[n] - x[n-2]$$

Then we read off the filter coefficients and define the vectors aa and bb as follows:

```
aa = [ 1, -0.9, 0.81 ]
bb = [ 1, 0, -1 ]
```

The following call to `freqz` generates a vector `HH` containing the values of the frequency response evaluated at the vector of frequencies specified by the third argument, `[-pi:(pi/100):pi]`:

```
HH = freqz(bb, aa, [-pi:(pi/100):pi])
```

A plot of the resulting magnitude and phase is shown in Fig. **??**. Since $H(e^{j\hat{\omega}})$ is always periodic with a period of 2π, it is sufficient to make the frequency response plot over the range $-\pi < \hat{\omega} \le \pi$.

In this example, we can show a connection between the poles and zeros and the shape of the frequency response. For this system, we have the system function

$$H(z) = \frac{1 - z^{-2}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

which has its poles at $z = 0.9e^{\pm j\pi/3}$ and zeros at $z = 1$ and $z = -1$. Since $z = -1$ is the same as $z = e^{j\pi}$, we conclude that $H(e^{j\hat{\omega}})$ is zero at $\hat{\omega} = \pi$, because $H(z) = 0$ at $z = -1$; likewise, the zero of $H(z)$ at $z = +1$ is a zero of $H(e^{j\hat{\omega}})$ at $\hat{\omega} = 0$. The poles have angles of $\pm\pi/3$ rad, so the poles have an effect on the frequency response near $\hat{\omega} = \pm\pi/3$. Since $H(z)$ is infinite at $z = 0.9e^{\pm j\pi/3}$, the nearby points on the unit circle (at $z = e^{\pm j\pi/3}$) must have large values. In this case, we can evaluate the frequency response directly from the formula to get

$$\begin{aligned}
H(e^{j\hat{\omega}})\big|_{\hat{\omega}=\pi/3} &= H(z)\big|_{z=e^{j\pi/3}} \\
&= \left.\frac{1 - z^{-2}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right|_{z=e^{j\pi/3}} \\
&= \frac{1 - (-\frac{1}{2} - j\frac{1}{2}\sqrt{3})}{1 - 0.9(\frac{1}{2} - j\frac{1}{2}\sqrt{3}) + 0.81(-\frac{1}{2} - j\frac{1}{2}\sqrt{3})} \\
\Rightarrow \left|H(e^{j\pi/3})\right| &= \frac{\left|1.5 + j0.5(\sqrt{3})\right|}{\left|0.145 + j0.045(\sqrt{3})\right|} = 10.522
\end{aligned}$$

This value of the frequency response magnitude is a good approximation to the true maximum value of 10.5257, which occurs at $\hat{\omega} = 0.334\pi$.

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