Example 10-4: Consider the first-order difference equation where the lone feedback coefficient is $a_1 = 1$,

$$y[n] = y[n-1] + x[n] \tag{10.3}$$

This system is often called an *accumulator system* because it simply adds the current sample of the input, $x[n]$, to the total of previous samples, $y[n-1]$. The impulse response of this system can be shown by iteration (**??**) to be the unit-step signal $h[n] = u[n]$. From the z-transform pair in (**??**) with $a = 1$, it follows that the system function is

$$H(z) = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1 - z^{-1}} \tag{10.4}$$

where the associated ROC for the infinite sum is $1 < |z|$.

Applying the condition for stability in (**??**) to $h[n] = u[n]$, we conclude that this is NOT a stable system, because the absolutely summability test is

$$\sum_{n=0}^{\infty} |u[n]| = \sum_{n=0}^{\infty} 1 \rightarrow \infty$$

Thus, it must be true that there is some bounded input that produces an unbounded output. One such example is a shifted step input $x[n] = u[n-1]$, for which the bound is $M_x = 1$. The result of Exercise **??** is that the output $y[n] = nu[n-1]$ grows linearly with n, so we cannot find a constant M_y such that $|y[n]| < M_y$ for all n.

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.