Example 3-10: Average Power in a Sinusoid. For the case where $x(t) = A\cos((2\pi/T_0)t + \varphi)$, the average power integral

$$E = \frac{1}{T_0}\int_0^{T_0} |A\cos(2\pi t/T_0 + \varphi)|^2 dt$$

can be evaluated directly. Although it is possible to use a trigonometric identity, we will use Euler's formula followed by the complex number identity $|z + w|^2 = |z|^2 + 2\Re\{zw^*\} + |w|^2$.

$$\begin{aligned}
E_T &= \frac{1}{T_0}\int_0^{T_0} \left|\tfrac{1}{2}Ae^{j((2\pi/T_0)t+\varphi)} + \tfrac{1}{2}Ae^{-j((2\pi/T_0)t+\varphi)}\right|^2 dt \\
&= \frac{1}{T_0}\int_0^{T_0} |\tfrac{1}{2}A|^2 + 2\Re\left\{\tfrac{1}{2}Ae^{j((2\pi/T_0)t+\varphi)}\tfrac{1}{2}Ae^{j((2\pi/T_0)t+\varphi)}\right\} + |\tfrac{1}{2}A|^2 dt \\
&= \frac{1}{T_0}\int_0^{T_0} \tfrac{1}{4}A^2 + 2\Re\left\{\tfrac{1}{4}A^2e^{j((4\pi/T_0)t+2\varphi)}\right\} + \tfrac{1}{4}A^2 dt \\
&= \frac{1}{T_0}\tfrac{1}{2}A^2T_0 + \frac{1}{T_0}\int_0^{T_0} \tfrac{1}{2}A^2\cos((4\pi/T_0)t + 2\varphi)dt \\
&= \tfrac{1}{2}A^2
\end{aligned}$$

Note that the integral of the cosine term is zero, being an integral over two complete periods.

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