Example 3-11: Demonstrate via MATLAB that summing the magnitude squared of the Fourier series coefficients (**??**) will converge to the average power in the square wave (**??**).

Solution: For the 50% duty cycle square wave, the average power is

$$E = \frac{1}{T_0}\int_0^{\frac{1}{2}T_0} |x(t)|^2 dt = \frac{1}{T_0}(1)^2 \tfrac{1}{2}T_0 = \tfrac{1}{2}$$

On the other hand, the Fourier series coefficients are $a_0 = \frac{1}{2}$ and

$$a_k = \frac{1}{j\pi k} \qquad\qquad k = \pm1, \pm3, \pm5, \ldots$$

Now let's start summing up the magnitude squared for $k = \pm1$, then $k = \pm3$, then $k = \pm5$, and so on.

$$\sum_{k=-1}^{1} |a_k|^2 = |a_0|^2 + |a_{-1}|^2 + |a_1|^2 = \tfrac{1}{4} + \frac{1}{\pi^2} + \frac{1}{\pi^2} = \tfrac{1}{4} + \frac{2}{\pi^2} \approx 0.4526$$

Now we include the $k = \pm3$ terms:

$$\sum_{k=-3}^{3} |a_k|^2 = |a_0|^2 + |a_{-1}|^2 + |a_1|^2 + |a_{-3}|^2 + |a_3|^2 = \tfrac{1}{4} + \frac{2}{\pi^2} + \frac{2}{9\pi^2} \approx 0.4752$$

Now we include the $k = \pm5$ terms:

$$\sum_{k=-5}^{5} |a_k|^2 = \tfrac{1}{4} + \frac{20}{9\pi^2} + \frac{1}{25\pi^2} + \frac{1}{25\pi^2} \approx 0.4833$$

Figure 3-1 shows that although the power converges slowly it already is within 5% of the final value of 0.5 with just the first and third harmonics, and within 2% if terms up to $|k| = 11$ are included.

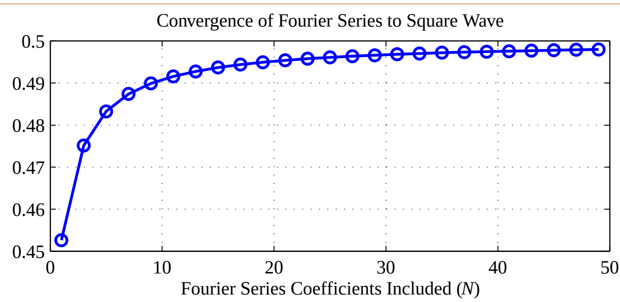

Figure 3-1: Convergence of the power in the Fourier series coefficients to the total power of a square wave. The average power of the square wave is $\frac{1}{2}$. The horizontal axis indicates the highest index Fourier series coefficient included in the synthesis summation (**??**).

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