Example 3-5: For the special case of a signal formed as the product of two sinusoids with frequencies $\frac{1}{2}$ Hz and 5 Hz

$$x(t) = \cos(\pi t)\sin(10\pi t) \tag{3.2}$$

it is necessary to rewrite $x(t)$ as a sum before its spectrum can be defined. One approach is to use the inverse Euler formula as follows:

$$x(t) = \left(\frac{e^{j\pi t} + e^{-j\pi t}}{2}\right)\left(\frac{e^{j10\pi t} - e^{-j10\pi t}}{2j}\right) \tag{3.3a}$$

$$= \tfrac{1}{4}e^{-j\pi/2}e^{j11\pi t} + \tfrac{1}{4}e^{-j\pi/2}e^{j9\pi t} + \tfrac{1}{4}e^{j\pi/2}e^{-j9\pi t} + \tfrac{1}{4}e^{j\pi/2}e^{-j11\pi t} \tag{3.3b}$$

$$= \tfrac{1}{2}\cos(11\pi t - \pi/2) + \tfrac{1}{2}\cos(9\pi t - \pi/2) \tag{3.3c}$$

In this derivation, we see four terms in the additive combination (3.3b), so there are four spectrum components at frequencies $\pm 11\pi$ and $\pm 9\pi$ rad/s, which convert to hertz as 5.5, 4.5, −4.5, and −5.5 Hz. The magnitude is the same ($\frac{1}{4}$) for all four components. It is also worth noting that neither of the originalfrequencies (5 Hz and $\frac{1}{2}$Hz) used to define $x(t)$ in (3.2) appear in the spectrum.

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.