Example 5-4: To illustrate the utility of the results that we have obtained for cascaded LTI systems, we derive a single system that is equivalent to the cascade of two systems defined by

$$h_1[n] = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & \text{otherwise} \end{cases} \qquad h_2[n] = \begin{cases} 1 & 1 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

The equivalent system, as in Fig. **??**(c), has an impulse response equal to $h[n] = h_1[n] * h_2[n]$. Therefore, to find the overall impulse response $h[n]$ we must convolve $h_1[n]$ with $h_2[n]$. This can be done by using the synthetic polynomial multiplication algorithm of Section **??**. In this case, the computation is as follows:

n	$n<0$	0	1	2	3	4	5	6	$n>6$
$h_1[n]$	0	1	1	1	1	0	0	0	0
$h_2[n]$	0	0	1	1	1				
$h_1[0]h_2[n]$	0	0	0	0	**0**	0	0	0	0
$h_1[1]h_2[n-1]$	0	0	1	1	**1**	1	0	0	0
$h_1[2]h_2[n-2]$	0	0	0	1	**1**	1	1	0	0
$h_1[3]h_2[n-3]$	0	0	0	0	**1**	1	1	1	0
$h[n]$	0	0	1	2	**3**	3	2	1	0

The equivalent impulse response in the bottom row also defines the filter coefficients $\{b_k\}$, and we can express $h[n]$ as a sum of weighted shifted impulses

$$h[n] = \sum_{k=0}^{6} b_k \delta[n-k]$$

where $\{b_k\}$ is the sequence $\{0, 1, 2, 3, 3, 2, 1\}$. The system with this impulse response $h[n]$ can be implemented either by the single difference equation

$$y[n] = \sum_{k=0}^{6} b_k x[n-k] \tag{5.1}$$

where $\{b_k\}$ is the above sequence, or by the pair of cascaded difference equations

$$\begin{aligned} v[n] &= \sum_{k=0}^{3} x[n-k] \\ y[n] &= \sum_{k=1}^{3} v[n-k] \end{aligned} \tag{5.2}$$

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.