

Example 6-10: Suppose that the first system in a cascade of two FIR LTI systems is defined by the set of coefficients $\{2, 1, 2\}$ and the second system is defined by the coefficients $\{0, 3, 0, -3\}$. Therefore, the frequency responses of the individual systems are

$$H_1(e^{j\hat{\omega}}) = 2 + e^{-j\hat{\omega}} + 2e^{-j\hat{\omega}2}$$

and

$$H_2(e^{j\hat{\omega}}) = 3e^{-j\hat{\omega}} - 3e^{-j\hat{\omega}3}$$

The overall frequency response is obtained by carrying out the multiplication of terms:

$$\begin{aligned} H(e^{j\hat{\omega}}) &= H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}}) \\ &= (2 + e^{-j\hat{\omega}} + 2e^{-j\hat{\omega}2}) (3e^{-j\hat{\omega}} - 3e^{-j\hat{\omega}3}) \\ &= 6e^{-j\hat{\omega}} + 3e^{-j\hat{\omega}2} + (6 - 6)e^{-j\hat{\omega}3} - 3e^{-j\hat{\omega}4} - 6e^{-j\hat{\omega}5} \end{aligned}$$

Thus, the overall equivalent impulse response is

$$h[n] = 6\delta[n - 1] + 3\delta[n - 2] - 3\delta[n - 4] - 6\delta[n - 5]$$

and the filter coefficients of the equivalent FIR system are $\{0, 6, 3, 0, -3, -6\}$.

