Example 6-11: Suppose that the first system in a cascade of two systems is defined by the set of coefficients $\{2, 4, 4, 2\}$ and the second system is defined by the coefficients $\{1, -2, 1\}$. The frequency responses of the individual systems are

$$H_1(e^{j\hat{\omega}}) = 2 + 4e^{-j\hat{\omega}} + 4e^{-j\hat{\omega}2} + 2e^{-j\hat{\omega}3}$$

and

$$H_2(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2}$$

The overall frequency response is

$$\begin{aligned} H(e^{j\hat{\omega}}) &= H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}}) \\ &= \left(2+4e^{-j\hat{\omega}}+4e^{-j\hat{\omega}2}+2e^{-j\hat{\omega}3}\right)\left(1-2e^{-j\hat{\omega}}+e^{-j\hat{\omega}2}\right) \\ &= 2 + 0e^{-j\hat{\omega}} - 2e^{-j\hat{\omega}2} - 2e^{-j\hat{\omega}3} + 0e^{-j\hat{\omega}4} + 2e^{-j\hat{\omega}5} \end{aligned}$$

Thus, the overall equivalent impulse response is

$$h[n] = 2\delta[n] - 2\delta[n-2] - 2\delta[n-3] + 2\delta[n-5]$$

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.