Example 6-12: Suppose that the first system in a cascade of two systems is defined by the set of coefficients $\{2, 4, 6, 4, 2\}$ and the second system is defined by the coefficients $\{1, -2, 2, -1\}$. The frequency responses of the individual systems are

$$H_1(e^{j\hat{\omega}}) = 2 + 4e^{-j\hat{\omega}} + 6e^{-j\hat{\omega}2} + 4e^{-j\hat{\omega}3} + 2e^{-j\hat{\omega}4}$$

and

$$H_2(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j\hat{\omega}2} - e^{-j\hat{\omega}3}$$

The overall frequency response is

$$\begin{aligned} H(e^{j\hat{\omega}}) &= H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}}) \\ &= \left(2 + 4e^{-j\hat{\omega}} + 6e^{-j\hat{\omega}2} + 4e^{-j\hat{\omega}3} + 2e^{-j\hat{\omega}4}\right)\left(1 - 2e^{-j\hat{\omega}} + 2e^{-j\hat{\omega}2} - e^{-j\hat{\omega}3}\right) \\ &= 2 + 0e^{-j\hat{\omega}} + 2e^{-j\hat{\omega}2} - 2e^{-j\hat{\omega}3} + 2e^{-j\hat{\omega}4} - 2e^{-j\hat{\omega}5} + 0e^{-j\hat{\omega}6} - 2e^{-j\hat{\omega}7} \end{aligned}$$

Thus, the overall equivalent impulse response is

$$h[n] = 2\delta[n] + 2\delta[n-2] - 2\delta[n-3] + 2\delta[n-4] - 2\delta[n-5] - 2\delta[n-7]$$

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