Example 6-3: For the FIR filter with coefficients $\{b_k\} = \{1, 2, 1\}$, find the output when the input is

$$x[n] = 3\cos\left(\frac{\pi}{3}n - \frac{\pi}{2}\right)$$

The frequency response of the system was determined in Example 6-1 to be

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

Note that $H(e^{-j\hat{\omega}}) = H^*(e^{j\hat{\omega}})$, so $H(e^{j\hat{\omega}})$ has conjugate symmetry. Solution of this problem requires just one evaluation of $H(e^{j\hat{\omega}})$ at the frequency $\hat{\omega} = \pi/3$:

$$\begin{aligned} H(e^{j\pi/3}) &= e^{-j\pi/3}\,(2 + 2\cos(\pi/3)) \\ &= e^{-j\pi/3}\left(2 + 2(\tfrac{1}{2})\right) = 3e^{-j\pi/3} \end{aligned}$$

Therefore, the magnitude is $|H(e^{j\pi/3})| = 3$ and the phase is $\angle H(e^{j\pi/3}) = -\pi/3$, so the output is

$$\begin{aligned} y[n] &= (3)(3)\cos\left(\frac{\pi}{3}n - \frac{\pi}{3} - \frac{\pi}{2}\right) \\ &= 9\cos\left(\frac{\pi}{3}(n-1) - \frac{\pi}{2}\right) \end{aligned}$$

Notice that the magnitude of the frequency response multiplies the amplitude of the cosine signal, and the phase angle of the frequency response adds to the phase of the cosine signal. This problem could also be studied and solved with the `dltidemo` MATLAB GUI.

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