Example 6-4: For the FIR filter with coefficients $\{b_k\} = \{1, 2, 1\}$, find the output when the input is

$$x[n] = 4 + 3\cos\left(\frac{\pi}{3}n - \frac{\pi}{2}\right) + 3\cos\left(\frac{7\pi}{8}n\right) \tag{6.1}$$

There are three frequencies, 0, $\pi/3$, and $7\pi/8$, so the output has three sinusoids at those same frequencies. The frequency response of the system was determined in Example 6-1, and is the same as the frequency response in Example 6-3. The input in this example differs from that of Example 6-3 by the addition of a constant (DC) term and an additional cosine signal of frequency $7\pi/8$. The solution by superposition therefore requires that we evaluate $H(e^{j\hat{\omega}})$ at frequencies 0, $\pi/3$, and $7\pi/8$, giving

$$\begin{aligned} H(e^{j0}) &= 4 \\ H(e^{j\pi/3}) &= 3e^{-j\pi/3} \\ H(e^{j7\pi/8}) &= 0.1522e^{-j7\pi/8} \end{aligned}$$

Therefore, the output is

$$\begin{aligned} y[n] &= 4 \cdot 4 + 3 \cdot 3\cos\left(\frac{\pi}{3}n - \frac{\pi}{3} - \frac{\pi}{2}\right) + 0.1522 \cdot 3\cos\left(\frac{7\pi}{8}n - \frac{7\pi}{8}\right) \\ &= 16 + 9\cos\left(\frac{\pi}{3}(n-1) - \frac{\pi}{2}\right) + 0.4567\cos\left(\frac{7\pi}{8}(n-1)\right) \end{aligned}$$

Notice that, in this case, the DC component is multiplied by 4, the component at frequency $\hat{\omega} = \pi/3$ is multiplied by 3, but the component at frequency $\hat{\omega} = 7\pi/8$ is multiplied by 0.1522. Because the frequency-response magnitude (gain) is so small at frequency $\hat{\omega} = 7\pi/8$, the component at this frequency is essentially *filtered out* of the input signal.

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