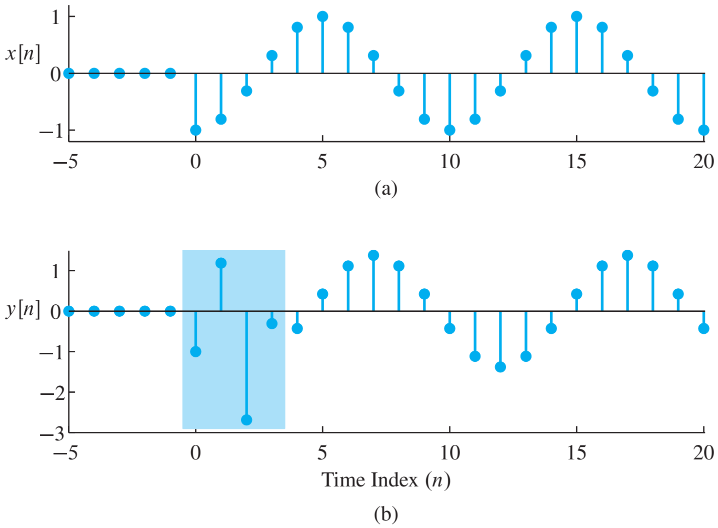

[!t]

Figure 6-1: (a) Input $x[n] = \cos(0.2\pi n - \pi)u[n]$ and (b) corresponding output $y[n]$ for the FIR filter with coefficients $\{1, -2, 4, -2, 1\}$. The transient region is the shaded area in (b). (Note the different vertical scales.)

Example 6-5: A simple example illustrates the above discussion. Consider the system of Exercise **??**, whose filter coefficients are the sequence $\{b_k\} = \{1, -2, 4, -2, 1\}$. The frequency response of this system is

$$H(e^{j\hat{\omega}}) = [4 - 4\cos(\hat{\omega}) + 2\cos(2\hat{\omega})]e^{-j2\hat{\omega}}$$

If the input is the suddenly applied cosine signal

$$x[n] = \cos(0.2\pi n - \pi)u[n]$$

we can represent it as the sum of two suddenly applied complex exponential signals. Therefore, the frequency response can be used as discussed in Section **??** to determine the corresponding steady-state output. Since $H(e^{j\hat{\omega}})$ at $\hat{\omega} = 0.2\pi$ is

$$H(e^{j\hat{\omega}}) = [4 - 4\cos(0.2\pi) + 2\cos(0.4\pi)]e^{-j2(0.2\pi)} = 1.382e^{-j(0.2\pi)2}$$

and $M = 4$, the steady-state output is

$$y[n] = 1.382\cos(0.2\pi(n-2) - \pi) \qquad 4 \leq n$$

The frequency response has allowed us to find a simple expression for the output everywhere in the steady-state region. If we desire the values of the output in the transient region, we could compute them using the difference equation for the system.

The input and output signals for this example are shown in Fig. 6-1. Since $M = 4$ for this system, the transient region is $0 \leq n \leq 3$ (indicated by the shaded region), and the steady-state region is $n \geq 4$. Also note that, as predicted by the steady-state analysis above, the signal in the steady-state region is simply a scaled (by 1.382) and shifted (by two samples) version of the input.

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.