Example 6-7: Often the frequency response is given by an equation containing sines or cosines, for example,

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}\,(3 - 2\cos\hat{\omega})$$

To obtain the FIR difference equation, it is necessary to write $H(e^{j\hat{\omega}})$ using powers of $e^{-j\hat{\omega}}$. In this example, we can use the inverse Euler formula $\cos\hat{\omega} = \frac{1}{2}(e^{j\hat{\omega}} + e^{-j\hat{\omega}})$, and then obtain

$$\begin{aligned} H(e^{j\hat{\omega}}) &= e^{-j\hat{\omega}}\left[3 - 2\left(\frac{e^{j\hat{\omega}} + e^{-j\hat{\omega}}}{2}\right)\right] \\ &= -1 + 3e^{-j\hat{\omega}} - e^{-j\hat{\omega}2} \end{aligned}$$

Now, it should be easy to see that $H(e^{j\hat{\omega}})$ corresponds to the following FIR difference equation:

$$y[n] = -x[n] + 3x[n-1] - x[n-2]$$

The impulse response, likewise, is easy to determine directly from $H(e^{j\hat{\omega}})$, when expressed in terms of powers of $e^{-j\hat{\omega}}$.

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