Example 6-8: Suppose that the input to a first-difference system is $x[n] = 4 + 2\cos(0.3\pi n - \pi/4)$. Since the output is related to the input by the difference equation $y[n] = x[n] - x[n-1]$, it follows that

$$\begin{aligned} y[n] &= 4 + 2\cos(0.3\pi n - \pi/4) - 4 - 2\cos(0.3\pi(n-1) - \pi/4) \\ &= 2\cos(0.3\pi n - \pi/4) - 2\cos(0.3\pi n - 0.55\pi) \end{aligned}$$

From this result, we see that the first-difference system removes the constant value and leaves two cosine signals of the same frequency, which could be combined by phasor addition. However, the solution using the frequency-response function is simpler. Since the first-difference system has frequency response

$$H(e^{j\hat{\omega}}) = 2\sin(\hat{\omega}/2)e^{j(\pi/2 - \hat{\omega}/2)}$$

the output of this system for the given input can be obtained more quickly via

$$y[n] = 4H(e^{j0}) + 2|H(e^{j0.3\pi})|\cos(0.3\pi n - \pi/4 + \angle H(e^{j0.3\pi}))$$

In other words, we need to evaluate the frequency response only at $\hat{\omega} = 0$ and $\hat{\omega} = 0.3\pi$. Since $H(e^{j0}) = 0$ and

$$\begin{aligned} H(e^{j0.3\pi}) &= 2\sin(0.3\pi/2)e^{j(0.5\pi - 0.15\pi)} \\ &= 0.908e^{j0.35\pi} \end{aligned}$$

the output is

$$\begin{aligned} y[n] &= (0.908)(2)\cos(0.3\pi n - \pi/4 + 0.35\pi) \\ &= 1.816\cos(0.3\pi n + 0.1\pi) \end{aligned}$$

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