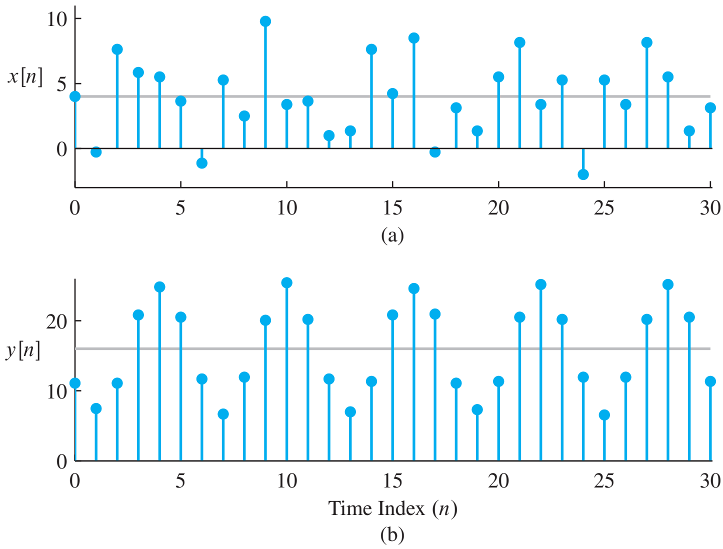

Figure 6-5: Input and output of a system with frequency response $H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$. (a) Segment of the input signal $x[n]$ given by (6.2), and (b) the corresponding segment of the output.

Example 6-9: If we repeat Example 6-4, we can show how the plot of $H(e^{j\hat{\omega}})$ in Fig. **??** makes it easy to find the filter's output for sinusoidal inputs. In Example 6-4, the input was

$$x[n] = 4 + 3\cos\left(\frac{\pi}{3}n - \frac{\pi}{2}\right) + 3\cos\left(\frac{7\pi}{8}n\right) \tag{6.2}$$

as shown in Fig. 6-5(a), and the filter coefficients were $\{b_k\} = \{1,\ 2,\ 1\}$. In order to get the output signal, we must evaluate $H(e^{j\hat{\omega}})$ at frequencies 0, $\pi/3$, and $7\pi/8$ giving

$$\begin{aligned} H(e^{j0}) &= 4 \\ H(e^{j\pi/3}) &= 3e^{-j\pi/3} \\ H(e^{j7\pi/8}) &= 0.1522e^{-j7\pi/8} \end{aligned}$$

These values are the points indicated with gray dots on the graphs of Fig. **??**. As in Example 6-4, the output is

$$\begin{aligned} y[n] &= 4\cdot 4 + 3\cdot 3\cos\left(\frac{\pi}{3}n - \frac{\pi}{3} - \frac{\pi}{2}\right) + 0.1522\cdot 3\cos\left(\frac{7\pi}{8}n - \frac{7\pi}{8}\right) \\ &= 16 + 9\cos\left(\frac{\pi}{3}(n-1) - \frac{\pi}{2}\right) + 0.4567\cos\left(\frac{7\pi}{8}(n-1)\right) \end{aligned}$$

We can see two features in the output signal $y[n]$. The sinusoid with $\hat{\omega} = 7\pi/8$ has a very small magnitude because the magnitude response around $\hat{\omega} = \pi$ is relatively small. Also, the linear phase slope of -1 means that the filter introduces a time delay of one sample which is evident in the second and third terms of $y[n]$.

The output of the simple lowpass filter is the time waveform shown in Fig. 6-5(b). Note that the DC component is indicated in both parts of the figure as a gray horizontal line. The output appears to be the sum of a constant level of 16 plus a cosine that has amplitude 9 and seems to be periodic with period 6. Closer inspection reveals that this is not exactly true because there is a third output component at frequency $\hat{\omega} = 7\pi/8$, which is just barely visible in Fig. 6-5(b). Its size is about 5% of the size of the component with frequency $\hat{\omega} = \pi/3$.