

**Example 7-4:** The delay property is a special case of the convolution property of the DTFT. To see this, recall that we can represent delay as the convolution with a shifted impulse

$$y[n] = x[n] * \delta[n - n_d] = x[n - n_d]$$

so the impulse response of a delay system is  $h[n] = \delta[n - n_d]$ . The corresponding frequency response (i.e., DTFT) of the delay system is

$$H(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} \delta[n - n_d] e^{-j\hat{\omega}n} = e^{-j\hat{\omega}n_d}$$

Therefore, using the convolution property, the DTFT of the output of the delay system is

$$Y(e^{j\hat{\omega}}) = X(e^{j\hat{\omega}})H(e^{j\hat{\omega}}) = X(e^{j\hat{\omega}})e^{-j\hat{\omega}n_d}$$

which is identical to the delay property of (??).

