Example 8-10: We might expect the fundamental period of a periodic signal to be the inverse of its fundamental frequency because this is true for continuous-time signals. However, for discrete-time signals this fact is often not true. The reason for this uncertainty is that *the period of the discrete-time signal must be an integer.*

Consider the signal $\tilde{x}_1[n] = \cos(0.125\pi n)$, whose frequency is $\pi/8$ rad. The period of this signal is $N = 16$; it is also the shortest period so we want to call 16 the fundamental period. If we take the 16-point DFT of one period of $\tilde{x}_1[n]$, we get $X_1[k] = 8\delta[k-1] + 8\delta[k-15]$. Then we can convert these DFT coefficients into a DFS representation with $a_1 = 8/16 = \frac{1}{2}$ and $a_{-1} = 8/16 = \frac{1}{2}$

$$\tilde{x}_1[n] = \tfrac{1}{2}e^{j(2\pi/16)n} + \tfrac{1}{2}e^{j(2\pi(-1)/16)n}$$

Now, consider the signal $\tilde{x}_2[n] = \cos(0.625\pi n)$, whose frequency is $5\pi/8$ rad; its period is not $2\pi/(5\pi/8) = 16/5$. Its period is also $N = 16$, and this is the shortest *integer* period. If we take the 16-point DFT of one period of $\tilde{x}_2[n]$, we get $X_2[k] = 8\delta[k-5] + 8\delta[k-11]$. Then we can convert these DFT coefficients into a DFS representation with $a_5 = 8/16 = \frac{1}{2}$ and $a_{-5} = 8/16 = \frac{1}{2}$

$$\tilde{x}_2[n] = \tfrac{1}{2}e^{j(2\pi(5)/16)n} + \tfrac{1}{2}e^{j(2\pi(-5)/16)n}$$

Our problem is that the period of $\tilde{x}_2[n]$ being 16 implies that the fundamental frequency is $2\pi/16$ so a_5 should be the fifth harmonic, but the definition of the cosine $\tilde{x}_2[n]$ has only one frequency $10\pi/16$ which has to be the fundamental frequency. In fact, this inconsistency happens whenever we take the DFT of a sinusoid with frequency $2\pi k_0/N$, and the integer k_0 is not a factor of N.

Therefore, this example illustrates that it is impossible to define a simple *consistent* relationship between the fundamental period and fundamental frequency of a periodic discrete-time signal.