Example 8-11: Suppose that we are given the following continuous-time signal

$$x(t) = 2\cos(42\pi t + 0.5\pi) + 4\cos(18\pi t)$$

and we want to determine the Fourier Series representation of the resulting discrete-time signal. In particular, we would like to determine which Fourier Series coefficients are nonzero. We need a sampling rate that is an integer multiple of the fundamental frequency and is also greater than the Nyquist rate, which is 42 Hz. The two frequency components in $x(t)$ are at 9 Hz and 21 Hz, so the fundamental frequency is the least common divisor, which is $f_0 = 3$ Hz. We must pick $N > 14$ to satisfy the Nyquist rate condition, so for this example we use $f_s = 16 \times 3 = 48$ Hz.

The sum of sinusoids can be converted to a sum of complex exponentials,

$$x(t) = e^{j(42\pi t+0.5\pi)} + e^{-j(42\pi t+0.5\pi)} + 2e^{j(18\pi t)} + 2e^{-j(18\pi t)}$$

and then (**??**) can be employed to represent the sampled signal $\tilde{x}[n] = x(n/f_s)$ as

$$\tilde{x}[n] = e^{j(42\pi(n/48)+0.5\pi)} + e^{-j(42\pi(n/48)+0.5\pi)} + 2e^{j(18\pi(n/48))} + 2e^{-j(18\pi(n/48))} \tag{8.1}$$

The four discrete-time frequencies are $\hat{\omega} = \pm(42/48)\pi$ and $\pm(18/48)\pi$. In order to write (8.1) in the summation form of (**??**), we use $N = 16$. In (8.1), we want to emphasize the term $(2\pi/N) = (2\pi/16)$ in the exponents, so we write

$$\tilde{x}[n] = e^{j((2\pi/16)7n+0.5\pi)} + e^{-j((2\pi/16)7n+0.5\pi)} + 2e^{j((2\pi/16)3n)} + 2e^{-j((2\pi/16)3n)} \tag{8.2}$$

Now we can recognize this sum of four terms as a special case of (**??**) with $N = 16$ and $M = 7$ (i.e., the range of the sum is $-7 \le m \le 7$). The only nonzero Fourier coefficients in (**??**) are at those for $m = \pm 7, \pm 3$, and their values are $a_3 = 2$, $a_7 = e^{j0.5\pi} = j$, $a_{-3} = 2$, and $a_{-7} = e^{-j0.5\pi} = -j$.