Example 8-12: The importance of analyzing the frequency-domain window characteristics becomes clear when we consider signals that are composed of sums of sinusoids. If we have a signal with two frequencies

$$x[n] = A_1 \cos \hat{\omega}_1 n + A_2 \cos \hat{\omega}_2 n$$

where $\hat{\omega}_1 < \hat{\omega}_2$, then the windowed signal would be

$$x_w[n] = w[n]x[n] = A_1 w[n] \cos \hat{\omega}_1 n + A_2 w[n] \cos \hat{\omega}_2 n$$

and the corresponding DTFT would be

$$X_w(e^{j\hat{\omega}}) = \frac{A_1}{2} W(e^{j(\hat{\omega}-\hat{\omega}_1)}) + \frac{A_1}{2} W(e^{j(\hat{\omega}+\hat{\omega}_1)}) + \frac{A_2}{2} W(e^{j(\hat{\omega}-\hat{\omega}_2)}) + \frac{A_2}{2} W(e^{j(\hat{\omega}+\hat{\omega}_2)}) \tag{8.3}$$

If we want to compute samples of $X_w(e^{j\hat{\omega}})$ in order to estimate $\hat{\omega}_1$ and $\hat{\omega}_2$, the main lobes of the terms in (8.3) should not overlap. If the window is a Hann window of main lobe width $8\pi/L$, the main lobes do not overlap if $\hat{\omega}_2 - \hat{\omega}_1 > 8\pi/L$. In this case, we would obtain distinct peaks at $\pm\hat{\omega}_1$ and $\pm\hat{\omega}_2$ and the peak heights are nearly equal to $\frac{1}{2}A_1$ and $\frac{1}{2}A_2$. When there are two distinct peaks in the DTFT, we say that the two frequencies are *resolved*. The effect of window length on spectrum resolution is discussed further in Section **??**.

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