Example 8-2: The 4-point DFT in Example 8-1 is the sequence $X[k] = \{2,\ \sqrt{2}e^{-j\pi/4},\ 0,\ \sqrt{2}e^{j\pi/4}\}$. If we compute the 4-point IDFT of this $X[k]$, we should recover $x[n]$ when we apply the IDFT summation (**??**) for each value of $n = 0, 1, 2, 3$. As before, the exponents in (**??**) are all integer multiples of $\pi/2$ when $N = 4$.

$$
\begin{aligned}
x[0] &= \tfrac{1}{4}\left(X[0]e^{j0} + X[1]e^{j0} + X[2]e^{j0} + X[3]e^{j0}\right) \\
&= \tfrac{1}{4}\left(2 + \sqrt{2}e^{-j\pi/4} + 0 + \sqrt{2}e^{j\pi/4}\right) = \tfrac{1}{4}(2 + (1 - j) + 0 + (1 + j)) = 1 \\
x[1] &= \tfrac{1}{4}\left(X[0]e^{j0} + X[1]e^{j\pi/2} + X[2]e^{j\pi} + X[3]e^{j3\pi/2}\right) \\
&= \tfrac{1}{4}\left(2 + \sqrt{2}e^{j(-\pi/4+\pi/2)} + 0 + \sqrt{2}e^{j(\pi/4+3\pi/2)}\right) = \tfrac{1}{4}(2 + (1 + j) + (1 - j)) = 1 \\
x[2] &= \tfrac{1}{4}\left(X[0]e^{j0} + X[1]e^{j\pi} + X[2]e^{j2\pi} + X[3]e^{j3\pi}\right) \\
&= \tfrac{1}{4}\left(2 + \sqrt{2}e^{j(-\pi/4+\pi)} + 0 + \sqrt{2}e^{j(\pi/4+3\pi)}\right) = \tfrac{1}{4}(2 + (-1 + j) + (-1 - j)) = 0 \\
x[3] &= \tfrac{1}{4}\left(X[0]e^{j0} + X[1]e^{j3\pi/2} + X[2]e^{j3\pi} + X[3]e^{j9\pi/2}\right) \\
&= \tfrac{1}{4}\left(2 + \sqrt{2}e^{j(-\pi/4+3\pi/2)} + 0 + \sqrt{2}e^{j(\pi/4+9\pi/2)}\right) = \tfrac{1}{4}(2 + (-1 - j) + (-1 + j)) = 0
\end{aligned}
$$

Thus, we have verified that the length-4 signal $x[n] = \{1,\ 1,\ 0,\ 0\}$ can be recovered from its 4-point DFT coefficients, $X[k] = \left\{2,\ \sqrt{2}e^{-j\pi/4},\ 0,\ \sqrt{2}e^{j\pi/4}\right\}$.

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