

Example 8-6: In number theory, a consistent algebraic system can be defined using remainders with respect to a fixed integer N , called the modulus. Recall that any integer n can be written uniquely as $n = qN + r$, where the quotient q is an integer and the remainder r is nonnegative and less than the modulus N . We write $r = n \bmod N$ to denote the remainder of n modulo- N . For example, $(-2) \bmod 10$ is equal to 8 because $-2 = (-1)(10) + 8$.

For signal delay and convolution, we only need addition and subtraction of integer indices. Suppose that $N = 10$, and we want to add 7 and 6. The result for modulo-10 arithmetic is 3 because we do normal addition $(7 + 6) = 13$, and then reduce modulo-10, taking the positive remainder which is 3. For mod-10 arithmetic the remainder must always be a positive integer in the range 0 to $N - 1 = 9$. If we subtract 4 from 2, the result $(2 - 4) \bmod 10 = -2 \bmod 10 = 8$. When we count up modulo-10, the sequence is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 3, \dots\}$ because adding 1 to 9 gives 10, and $10 \bmod 10 = 0$. Furthermore, if we evaluate $(n - 4) \bmod 10$ for $n = 0, 1, \dots, 9$, we start at $(0 - 4) \bmod 10$ which is equal to 6 and get $\{6, 7, 8, 9, 0, 1, 2, 3, 4, 5\}$.

