Example 8-7: Suppose that $x[n]$ is a length-10 rectangular pulse, and $h[n]$ is a length-6 rectangular pulse. From Chapter 5, we know that the convolution result has a trapezoidal shape. If we use 10-point DFTs, it is possible to work out the answer analytically. The 10-point DFT of $x[n]$ is $X[k] = 10\delta[k]$, because $x[n]$ is all ones and we can use the DFT pair in (**??**) with $k_0 = 0$. Then we can carry out the multiplication of DFTs without knowing $H[k]$. Since $\delta[k]$ is nonzero only when $k = 0$, we obtain

$$Y[k] = X[k]H[k] = 10\delta[k]H[k] = 10H[0]\delta[k]$$

The value of $H[0]$ is the DFT coefficient at $k_0 = 0$, which is

$$H[0] = \sum_{n=0}^{9} h[n]\cancelto{1}{e^{-j0.2\pi(0)n}} = 6$$

Thus, we have $Y[k] = 60\delta[k]$ and we can take the length-10 IDFT to get $y[n]$

$$y[n] = \frac{1}{10}\sum_{k=0}^{9} 60\delta[k]e^{j0.2\pi kn} = \underbrace{6\delta[0]e^{j0}}_{k=0 \text{ term}} = 6 \quad \text{for } n = 0, 1, \ldots 9$$

The only nonzero term in the sum is the one for $k = 0$. The convolution result $y[n]$ is a constant which is not what we want.

If we change the length of the DFTs to $N = 16$, we can get the correct convolution of the two rectangular pulses where the length is $10 + 6 - 1 = 15$. This case cannot be done algebraically without a lot of tedious manipulations, so we would prefer to use MATLAB.

```
x = ones(1,10); h=[1,1,1,1,1,1];
y16 = ifft( fft( h, 16 ) .* fft( x, 16 ), 16);
y10 = ifft( fft( h, 10 ) .* fft( x, 10 ), 10);
```

The results for the $N = 16$ case are plotted in Fig. **??**, but we omit the plots for $N = 10$ because the signal $\tilde{y}[n]$ is constant.

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