Example 8-8: Suppose that a signal $\tilde{x}[n]$ is defined with a DFS summation like (**??**) with specific values for the coefficients $a_m = (m^2 - 1)$. If $M = 2$, then

$$\tilde{x}[n] = \sum_{m=-2}^{2} (m^2 - 1) e^{j(2\pi m/N)n} \qquad \text{for } -\infty < n < \infty$$

For the case $N = 5$, make a list of the values of $\tilde{x}[n]$ for $n = 0, 1, 2, \ldots, 10$ to show that $\tilde{x}[n]$ has a period equal to 5.

Solution: The summation formula for $\tilde{x}[n]$ can be written out

$$\begin{aligned}
\tilde{x}[n] &= ((-2)^2 - 1)e^{j(2\pi/5)(-2)n} + ((-1)^2 - 1)e^{j(2\pi/5)(-1)n} + ((0)^2 - 1)e^{j(2\pi/5)(0)n} \\
&\qquad + ((1)^2 - 1)e^{j(2\pi/5)(1)n} + ((2)^2 - 1)e^{j(2\pi/5)(2)n} \\
&= (4 - 1)e^{j(2\pi/5)(-2)n} + (-1)e^{j(2\pi/5)(0)n} + (4 - 1)e^{j(2\pi/5)(2)n} \\
&= 3e^{j(2\pi/5)(-2)n} - e^{j(2\pi/5)(0)n} + 3e^{j(2\pi/5)(2)n} \\
&= 6\cos(4\pi n/5) - 1
\end{aligned}$$

This expression can be evaluated by substituting integer values for n to obtain the following list of values for $0 \le n \le 10$:

$$\tilde{x}[n] = \{5,\ -5.045,\ 0.545,\ 0.545,\ -5.045,\ 5,\ -5.045,\ 0.545,\ 0.545,\ -5.045,\ 5\}$$

Thus, we see that $\tilde{x}[n]$ repeats with a period of 5.

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