Example 8-9: When determining the DFS coefficients $\{a_m\}$ from the DFT $X[k]$, there are two cases to consider: N even and N odd. The notation is much easier when N is odd because we can write $N = 2M + 1$ where M is an integer. For example, when $N = 5$ we have $M = 2$, so the Fourier Series would be

$$\tilde{x}[n] = a_0 + a_1 e^{j0.4\pi n} + a_2 e^{j0.8\pi n} + a_{-1} e^{-j0.4\pi n} + a_{-2} e^{-j0.8\pi n}$$

The 5-point DFT of one period of $\tilde{x}[n]$ is $\{Na_0, Na_1, Na_2, Na_{-2}, Na_{-1}\}$. On the other hand, when N is even there is a complication. For example, when $N = 4$ the summation in (**??**) implies that $2M \leq 3$, or $M = 1$, but using the 4-point IDFT we can write

$$\tilde{x}[n] = \frac{1}{N} X[0] + \frac{1}{N} X[1] e^{j0.5\pi n} + \frac{1}{N} X[2] e^{j\pi n} + \frac{1}{N} X[3] e^{j1.5\pi n}$$

so the DFS representation of $\tilde{x}[n]$ would be

$$\tilde{x}[n] = a_0 + a_1 e^{j0.5\pi n} + a_2 e^{j\pi n} + a_{-1} e^{-j0.5\pi n}$$

The case where $a_2 \neq 0$ is a special case that is similar to the ambiguity with $X[N/2]$ treated in Section **??**. The value of a_2 (or $X[N/2]$) is real when the signal is real, so it does not require a complex conjugate term in negative frequency.

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