Example 9-10: The system function studied in Example 9-7 is $H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$, and its zeros are

$$
\begin{aligned}
z_1 &= 1 & &\Rightarrow H(z_1) = 0 \\
z_2 &= \tfrac{1}{2} + j\tfrac{1}{2}\sqrt{3} \;=\; 1e^{j\pi/3} & &\Rightarrow H(z_2) = 0 \\
z_3 &= \tfrac{1}{2} - j\tfrac{1}{2}\sqrt{3} \;=\; 1e^{-j\pi/3} & &\Rightarrow H(z_3) = 0
\end{aligned}
$$

As shown in Fig. 9-8, these zeros all lie on the unit circle. Since the angle of a vector from the origin to a point on the unit circle corresponds to $\hat{\omega}$ in the frequency domain, complex sinusoids with frequencies 0, $\pi/3$, and $-\pi/3$ are "nulled out" by the system. That is, the output resulting from each of the following three signals will be zero:

$$
\begin{aligned}
x_1[n] = (z_1)^n &= 1 \\
x_2[n] = (z_2)^n &= e^{j(\pi/3)n} \\
x_3[n] = (z_3)^n &= e^{-j(\pi/3)n}
\end{aligned}
$$

For example, when the input is $x_2[n]$, the output is $y_2[n] = H(z_2)(z_2)^n = 0$.

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