Example 9-11: The 11-point running-sum filter ($L = 11$) is the special case where $a = 1$ in (**??**), so one way to write its system function is

$$H(z) = \sum_{k=0}^{10} z^{-k} = \frac{1 - z^{-11}}{1 - z^{-1}} \tag{9.3}$$

Then the frequency response is obtained by substituting $e^{j\hat{\omega}}$ for z, followed by reducing the rational form to a Dirichlet form as was done in (**??**) and (**??**) of Chapter **??**

$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} = \frac{1 - e^{-j11\hat{\omega}}}{1 - e^{-j\hat{\omega}}} = D_{11}(\hat{\omega})e^{-j5\hat{\omega}} \tag{9.4}$$

The pole-zero diagram for this case was shown previously in Fig. **??**, along with the corresponding frequency response in Fig. **??**. The 11 factors of the numerator polynomial $(z^{11} - 1)$ in (9.3) include a term $(z - 1)$ for the zero at $z = 1$ which is canceled by the corresponding term $(z - 1)$ in the denominator. This explains why we have only ten zeros around the unit circle with the gap at $z = 1$. The ten zeros around the unit circle in Fig. **??** show up as zeros of $H(e^{j\hat{\omega}})$ along the $\hat{\omega}$ axis in Fig. **??** at integer multiples of $2\pi/11$, except for $\hat{\omega} = 0$. Note that it is the gap at $z = 1$ that allows the frequency response to be larger at $\hat{\omega} = 0$. The other zeros around the unit circle keep $H(e^{j\hat{\omega}})$ small, thereby creating the "lowpass" filter frequency response shown in Fig. **??**.

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