Example 9-7: Consider the following third-order system

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

which we want to split into the cascade of lower order systems with real coefficients. Since we want a product of system functions for the cascade, we need to find the roots and use the product form in (**??**). The MATLAB function `roots([1, -2, 2, -1])` will extract the roots giving

```
1.0,  0.5000 + 0.8660i, and 0.5000 - 0.8660i
```

One of the roots of $H(z)$ is $z = 1$, so $H_1(z) = (1 - z^{-1})$ is a factor of $H(z)$. The other two first-order factors have complex roots, but these roots are complex conjugates. Therefore, in order to obtain the other system function for the cascade we must form the product of these terms as

$$H_2(z) = (1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1}) = 1 - z^{-1} + z^{-2}$$

The resulting second-order factor has real coefficients. This is an instance of the general property that the complex-valued roots of a polynomial with real coefficients must occur in conjugate pairs. The factorization of $H(z)$ as

$$H(z) = (1 - z^{-1})(1 - z^{-1} + z^{-2})$$

gives the cascade shown in the block diagram of Fig. **??**. The resulting difference equations for the cascade are obtained by inverse transforming the factors of $H(z)$

$$w[n] = x[n] - x[n-1]$$
$$y[n] = w[n] - w[n-1] + w[n-2]$$