Example C-3: For pulse waves, we have already derived a formula (**??**) for the Fourier coefficients, a_k, and we can easily show that the time-scaling property is true for all pulse signals with the same ratio between pulse duration and period, τ/T_0. The formula (**??**) for the Fourier coefficients of a pulse wave is repeated below

$$a_k = \begin{cases} \dfrac{\sin(\pi k(\tau/T_0))}{\pi k} & k = \pm 1, \pm 2, \pm 3, \ldots \\ \tau/T_0 & k = 0 \end{cases}$$

Since τ and T_0 always appear together as the ratio τ/T_0, the a_k coefficients depend only on the ratio.

For example, when the period is 50 ms and the pulse width is 10 ms, the ratio $\tau/T_0 = 0.2$. Thus, we can determine the Fourier coefficients for all cases where $\tau/T_0 = 0.2$ by using the Fourier analysis integral for a 0.2 s pulse repeated with a period of 1 s. The following integral is for a pulse length of τ' with a period of 1 s

$$a_k = \int_{-\tau'/2}^{\tau'/2} e^{-j2\pi k t}\, dt$$

so the limits on the integral are $\pm\tau'/2 = \pm(\tau/T_0)/2 = \pm 0.1$ for this case. This integral has fewer symbols to complicate the integration and subsequent algebraic manipulations. It would be ideal for use with symbolic analysis programs such as Wolfram|Alpha.

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