

Example C-5: Consider a triangular wave $x(t)$ as depicted in the plot of Fig. ??(a). The derivative is the slope of the linear segments, but is discontinuous at $t = 0, 0.02, 0.04, \dots$; hence, the derivative is a square wave as shown in Fig. ??(b). In general, the Fourier series coefficients a_k for the triangular wave $x(t)$ are given by (??). When the period is $T_0 = 0.04$ s, $\omega_0 = 2\pi/(0.04) = 50\pi$, so the Fourier series coefficients of the differentiated signal $y(t)$ are $b_k = (j50\pi k)a_k$, or

$$b_k = (j50\pi k)a_k = \begin{cases} 50 \frac{(-2j)}{\pi k} & k = \pm 1, \pm 3, \pm 5, \dots \\ 0 & k = \pm 2, \pm 4, \pm 6, \dots \\ 0 & k = 0 \end{cases} \quad (\text{C.2})$$

These are the Fourier series coefficients for a square wave that varies between +50 and -50. The DC value is zero because the multiplier $(j50\pi k)$ is zero for $k = 0$. The plot in Fig. ??(c) was constructed using (C.2) and the finite synthesis sum (??) with $N = 35$. Like the pulse waveform synthesis in Fig. ??, we see ripples near the discontinuities. Away from the points of discontinuous slope, however, the approximation converges to +50 and -50, which are the slopes of the triangular wave in the first and last half of the period, respectively.

