EXERCISE 8.1: *Orthogonality Property of Periodic Discrete-Time Complex Exponentials*

Use the formula

$$\sum_{k=0}^{N-1} \alpha^k = \frac{1-\alpha^N}{1-\alpha} \tag{8.8}$$

to show that

$$\begin{aligned}
d[n-m] &= \frac{1}{N}\sum_{k=0}^{N-1} e^{j(2\pi/N)nk} e^{j(2\pi/N)(-m)k} && \text{(definition)} \\
&= \frac{1}{N}\sum_{k=0}^{N-1} e^{j(2\pi/N)(n-m)k} && \text{(alternate form)} \\
&= \frac{1}{N}\left(\frac{1-e^{j(2\pi)(n-m)}}{1-e^{j(2\pi/N)(n-m)}}\right) && \text{(use (8.8))}
\end{aligned}$$

$$d[n-m] = \begin{cases} 1 & n-m = rN \\ 0 & \text{otherwise} \end{cases} \tag{8.9}$$

where r is any positive or negative integer including $r = 0$.

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.