**LECTURE OBJECTIVES**

- **Operations** on a time-domain signal $x(t)$ have a **SIMPLE form** in the frequency-domain.

- **SPECTRUM** Representation has lines at: $(A_k, \varphi_k, f_k)$

- Represents Sinusoid with **DIFFERENT** Frequencies

$$x(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \varphi_k)$$

**Recall FREQUENCY DIAGRAM**

- Used to visualize relationship between frequencies, amplitudes and phases
- Plot Complex Amplitude vs. Freq
GRAPHICAL SPECTRUM

\[ -2\sin(7t + 0.1\pi) = \frac{1}{2}e^{j\pi}e^{-j0.5\pi}e^{j0.1\pi}e^{j7t} + \frac{1}{2}e^{-j\pi}e^{j0.5\pi}e^{-j0.1\pi}e^{-j7t} = e^{j0.6\pi}e^{j7t} + e^{-j0.6\pi}e^{-j7t} = 2\cos(7t + 0.6\pi) \]

AMPLITUDE, PHASE & FREQUENCY are shown

OPERATIONS on SPECTRUM

- Adding DC, or amplitude scaling
- Adding two (or more) signals
- Time-Shifting
  - Multiply in frequency by complex exponential
- Differentiation of \( x(t) \)
  - Multiply in frequency-domain by \((j\omega)\)
- Frequency Shifting
  - Multiply in time-domain by sinusoid

General Spectrum

- \(2M + 1\) spectrum components:
  \[ x(t) = \sum_{k=-M}^{M} a_k e^{j2\pi f_k t} \]

- At \( f = f_k \) the complex amplitude is \( a_k \)
  - usually, for real \( x(t) \) \( f_0 = 0 \)

Scaling or Adding a constant

- Adding DC
  \[ x(t) + c = \sum_{k\neq0} a_k e^{j2\pi f_k t} + a_0 e^{j2\pi(0)t} + ce^{j2\pi(0)t} \]
  - new DC is \( a_0 + c \)

- Scaling
  \[ \gamma x(t) = \gamma \sum_{k=-M}^{M} a_k e^{j2\pi f_k t} = \sum_{k=-M}^{M} (\gamma a_k) e^{j2\pi f_k t} \]
Scaling and Adding a constant

$$2x(t) + 6 = \sum_{k \neq 0} 2a_k e^{j2\pi f_k t} + 2a_0 + 6$$

Adding Two Signals (1)

- Adding signals with same fundamental

$$x_1(t) + x_2(t) = \sum_{k=-M}^{M} a_{1k} e^{j2\pi f_k t} + \sum_{k=-M}^{M} a_{2k} e^{j2\pi f_k t} = \sum_{k=-M}^{M} (a_{1k} + a_{2k}) e^{j2\pi f_k t}$$

Adding Two Signals (2)

- Adding signals with same fundamental

Time Shifting x(t)

- Time Shifting

$$x(t - \tau_d) = \sum_{k=-M}^{M} a_k e^{j2\pi f_k (t-\tau_d)} = \sum_{k=-M}^{M} (a_k e^{-j2\pi f_k \tau_d}) e^{j2\pi f_k t}$$

- Multiply Spectrum complex amplitudes by a complex exponential

$$y(t) = \sum_{k=-M}^{M} b_k e^{j2\pi f_k t}$$
Differentiating $x(t)$

- Take **derivative** of the Signal $x(t)$

$$\frac{d}{dt} x(t) = \sum_{k=-M}^{M} a_k (j2\pi f_k) e^{j2\pi f_k t} = \sum_{k=-M}^{M} (j2\pi f_k) a_k e^{j2\pi f_k t}$$

$$y(t) = \sum_{k=-M}^{M} b_k e^{j2\pi f_k t}$$

- Multiply complex amplitudes by “$j\omega$” = “$j2\pi f$”

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Frequency Shifting $x(t)$

- Multiply $x(t)$ by Complex Exponential

$$y(t) = A e^{j\varphi} e^{j2\pi f_c t} x(t)$$

$$y(t) = \sum_{k=-M}^{M} A e^{j\varphi} e^{j2\pi f_c t} a_k e^{j2\pi f_k t}$$

$$= \sum_{k=-M}^{M} (a_k A e^{j\varphi}) e^{j2\pi (f_k + f_c) t}$$

- Spectrum components shifted: $f_k \rightarrow f_k + f_c$