PROBLEM:

Complex exponentials obey the expected rules of algebra when doing integrals and derivatives. Consider the complex signal $z(t) = Ze^{j\pi t/2}$ where $Z = e^{-j\pi/3}$.

(a) Show that the first derivative of $z(t)$ with respect to time can be represented as a new complex exponential $Qe^{j\pi t/2}$, i.e., $\frac{d}{dt}z(t) = Qe^{j\pi t/2}$. Determine the value for the complex amplitude Q. How much greater (or smaller) is the angle of Q than the angle of Z.

(b) Evaluate the definite integral of $z(t)$ over the range $0 \le t \le 1$: $\displaystyle\int_0^1 z(t)dt = ?$

Note that integrating a complex quantity follows the expected rules of algebra: you could integrate the real and imaginary parts separately, but you can also *use the integration formula for an exponential* directly on $z(t)$.

(c) Evaluate the integral of the magnitude squared $|z(t)|^2$ over the range $-1 \le t \le 1$:

$$\int_{-1}^{1} |z(t)|^2 dt = ?$$