PROBLEM:

A computer music student decided that he would create a new type of signal where the frequency moved as a function of time. In particular, he planned to generate a "chirp" signal—one that swept in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi$ as time goes from $t = 0$ to $t = T_2$.

It turns out that the *instantaneous frequency* of the chirp can be derived from a derivative operation. If we define $x(t)$ in the following manner:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

then we can think of this as the cosine function operating on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp.

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians} \tag{2}$$

(a) For the "chirp" in (1) determine formulas for the beginning frequency (ω_1) and ending frequency (ω_2) in terms of α, β and T_2.

(b) For the "chirp" signal

$$x(t) = \Re e\{e^{j(40t^2 + 27t + 13)}\}$$

derive a formula for the *instantaneous* frequency.

(c) Make a plot of the *instantaneous* frequency (in Hertz) versus time over the range $0 \le t \le 1$ sec.