PROBLEM:

The diagram in Figure 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

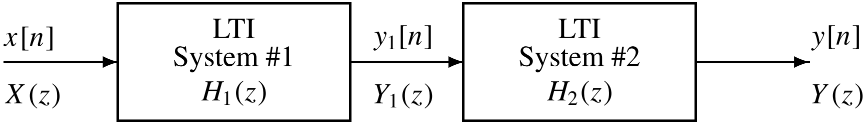

Figure 1: Cascade connection of two LTI systems.

(a) Use z-transforms to show that the system function for the system function of the overall system (from $x[n]$ to $y[n]$) is $H(z) = H_2(z)H_1(z)$, where $Y(z) = H(z)X(z)$.

(b) Derive a condition on $H(z)$ that guarantees that the output signal will always be equal to the input signal.

(c) Suppose that System #1 is an FIR filter described by the difference equation $y_1[n] = x[n] + \frac{5}{6}x[n-1]$ and System #2 is described by the system function $H_2(z) = 1 - 2z^{-1} + z^{-2}$. Determine the system function of the overall cascade system.

(d) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Figure 1.

(e) Plot the poles and zeros of $H(z)$ in the complex z-plane.

(f) If System #1 is the difference equation: $y_1[n] = x[n] + \frac{5}{6}x[n-1]$, find a system function $H_2(z)$ so that output of the cascaded system will always be equal to its input. In other words, find $H_2(z)$ which will undo the filtering action of $H_1(z)$. This is called *deconvolution*.