PROBLEM:

A periodic signal $x(t) = x(t + T_0)$ is described over one period $-T_0/2 \leq t \leq T_0/2$ by the equation

$$x(t) = \begin{cases} 1 & |t| < t_c \\ 0 & t_c < |t| \leq T_0/2 \end{cases}$$

where $t_c < T_0/2$.

(a) Sketch the periodic function $x(t)$ for $-T_0 < t < 2T_0$ for the case $t_c = T_0/8$.

(b) Determine the D.C. coefficient X_0. (This answer will depend on t_c and T_0.)

(c) Use the Fourier *analysis* integral (for $k \neq 0$)

$$X_k = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\omega_0 t} dt$$

to determine a general formula for the Fourier coefficients X_k in the representation

$$x(t) = X_0 + \Re e \left\{ \sum_{k=1}^{\infty} X_k e^{jk\omega_0 t} \right\}$$

Your final result should depend on t_c and T_0.
Notes: the frequency ω_0 is given in radians/sec. The integral can be done over any period of the signal; in this case, the most convenient choice is from $-T_0/2$ to $T_0/2$.

(d) Sketch the spectrum of $x(t)$ for the case $\omega_0 = 2\pi(100)$ rad/sec and $t_c = T_0/4$ for frequencies between $-5\omega_0$ and $+5\omega_0$. Label each component with its complex amplitude (magnitude and phase).

(e) Compare your answer in part (d) to the formula for the Fourier coefficients of a 50% duty cycle square wave given in class (and in equation (3.4.5) in the book). Compare both the magnitudes and phases of X_k, as well as the trend versus k. State the similarities and also the differences.

(f) Sketch the spectrum of $x(t)$ for the case $\omega_0 = 2\pi(100)$ rad/sec and $t_c = T_0/8$ for frequencies between $-5\omega_0$ and $+5\omega_0$. Label each component with its complex amplitude (magnitude and phase).