PROBLEM:

Consider a system defined by

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

(a) Suppose that the input $x[n]$ is non-zero only for $0 \leq n \leq N-1$; i.e., it has a support of N samples. Show that $y[n]$ is non-zero at most over a finite interval of the form $0 \leq n \leq P-1$. Determine P and the support of $y[n]$ in terms of M and N.

(b) Suppose that the input $x[n]$ is non-zero only for $N_1 \leq n \leq N_2$. What is the support of $x[n]$? Show that $y[n]$ is non-zero at most over a finite interval of the form $N_3 \leq n \leq N_4$. Determine N_3 and N_4 and the support of $y[n]$ in terms of N_1, N_2, and M.

Hint: Draw a sketch similar to Fig. 5.5 to illustrate the zero regions of the output signal.