

PROBLEM:

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with A-440 and ending with A-880 are:

note name	A	B ^D	B	C	C [#]	D	E ^D	E	F	F [#]	G	G [#]	A
note number	49	50	51	52	53	54	55	56	57	58	59	60	61
frequency													

- (a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.
- (b) Make a table of the frequencies of the tones of the octave beginning with A-440 and ending at A-880. Recall that A-440 is the A above middle C (note #49) which is tuned to 440 Hz.
- (c) The notes (from part (b)) on a piano keyboard are numbered 49 through 61. If n denotes the note number, and f denotes the frequency of the corresponding tone in hertz, give a formula for the frequency of the tone as a function of the note number.
- (d) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The A Minor chord is composed of the tones of A, C, E sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the A-Minor chord assuming that each note is realized by a pure sinusoidal tone and that each note is equally loud. (You do not have to specify the complex amplitudes precisely.)