PROBLEM:

Use the signal $x(t)$ defined by the equation

$$x(t) = \begin{cases} t & 0 \le t \le t_c \\ 0 & t_c < t \le T_0 \end{cases}$$

where $t_c = \frac{1}{2}T_0$.

(a) Use the Fourier $\boxed{\textit{analysis}}$ integral (for $k \neq 0$)

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

to determine a general formula for the Fourier Series coefficients a_k. Your final result for a_k should depend on k.

Notes: This Fourier integral requires integration by parts; in addition, the Fourier integral can be done over any period of the signal; in this case, the most convenient choice is from 0 to T_0.

Note: the frequency ω_0 would be given in rads/sec, but it does not have a specific value. However, you can simplify your formulas by using the identity $\omega_0 T_0 = 2\pi$.

(b) Use the Fourier Series coefficients to sketch the spectrum of $x(t)$ for the case $\omega_0 = 2\pi(\frac{1}{4})$ rad/sec and $t_c = \frac{1}{2}T_0$. Include *only* those frequency components corresponding to $k = 0, \pm1, \pm2, \pm3$. Label each component with its frequency and its complex amplitude (i.e., numerical values of magnitude and phase).