PROBLEM:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the "angle" of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying angle argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the angle argument $\psi(t)$ is the *instantaneous frequency*, which is also the audible frequency heard from the chirp. (The instantaneous frequency is the frequency heard by the human ear when the chirp rate is relatively slow. There are cases of FM where the audible signal is quite different, but these happen when the chirp rate is very high.)

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

(a) For the "chirp" signal

$$x(t) = \Re e\left\{e^{j2\pi(-75t^2 + 900t + 33)}\right\}$$

derive a formula for the *instantaneous* frequency versus time.

(b) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 2$ sec.