PROBLEM:

Suppose that three systems are hooked together in "cascade." In other words, the output of $\mathcal{S}_1$ is the input to $\mathcal{S}_2$, and the output of $\mathcal{S}_2$ is the input to $\mathcal{S}_3$. The three systems are specified as follows:

$$\mathcal{S}_1: \qquad y_1[n] = x_1[n] + x_1[n-2]$$

$$\mathcal{S}_2: \qquad y_2[n] = 7x_2[n-5] + 7x_2[n-6]$$

$$\mathcal{S}_3: \qquad \mathcal{H}_3(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j2\hat{\omega}}$$

NOTE: the output of $\mathcal{S}_i$ is $y_i[n]$ and the input is $x_i[n]$.

The objective in this problem is to determine the equivalent system that is a single operation from the input $x[n]$ (into $\mathcal{S}_1$) to the output $y[n]$ which is the output of $\mathcal{S}_3$. Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

(a) Determine the difference equation for $\mathcal{S}_3$.

(b) Determine the frequency response of the first two systems: $\mathcal{H}_i(\hat{\omega})$ for $i = 1, 2$.

(c) Determine the frequency response of the overall cascaded system.

(d) Write *one difference equation* that defines the overall system in terms of $x[n]$ and $y[n]$ only.