PROBLEM:

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you know well that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Thus, the ratio must be $2^{1/12}$. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with middle-C and ending with high-C are:

note name	C	$C^\sharp$	D	$E^\flat$	E	F	$F^\sharp$	G	$G^\sharp$	$A(440)$	$B^\flat$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency										440			

(a) Make a table of the frequencies of the tones of the octave beginning with middle-C assuming that A above middle C is tuned to 440 Hz.

(b) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

(c) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The D Major chord is composed of the tones of $D \quad F^\sharp \quad A$ sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the D Major chord assuming that each note is realized by a pure sinusoidal tone. (You do not have to specify the complex phasors precisely.)