PROBLEM:

A periodic signal $x(t)$ is described over one period $0 \le t \le T_0$ by the equation

$$x(t) = \begin{cases} \dfrac{2t}{T_0} & 0 \le t < T_0/2 \\ 0 & T_0/2 \le t \le T_0. \end{cases}$$

We have seen that such a periodic signal can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{where} \quad a_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-jk\omega_0 t}\, dt$$

(a) Sketch the periodic function $x(t)$ for $-T_0 < t < 2T_0$.

(b) Determine a_0, the D.C. coefficient for the Fourier series.

(c) Set up the Fourier $\boxed{\textit{analysis}}$ integral for determining a_k for $k \neq 0$. (Insert proper limits and integrand.) What integration technique from calculus could you apply to aid in evaluating this integral?

You do not have to evaluate the integral in part (c). If you are curious, the answer is:

$$a_k = \begin{cases} \dfrac{(1 + j\pi k)e^{-j\pi k} - 1}{2\pi^2 k^2} = \dfrac{(1 + j\pi k)(-1)^k - 1}{2\pi^2 k^2} & k \neq 0 \\ \text{your value of } a_0 \text{ found in (b)} & k = 0 \end{cases}$$

Note: A similar problem is worked out in great detail in Section 33.5.3 of the Fourier Series Notes on WebCT.