PROBLEM:

A periodic signal $x(t)$ with a period $T_0 = 10$ is described *over one period,* $0 \le t \le 10$, by the equation

$$x(t) = \begin{cases} 0 & 0 \le t \le 5 \\ 2 & 5 < t \le 10. \end{cases}$$

This signal can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t},$$

which is valid for all time $-\infty < t < \infty$.

(a) Sketch the periodic function $x(t)$ for $-10 < t < 20$.

(b) Determine a_0, the D.C. coefficient of the Fourier Series.

(c) Use the Fourier $\boxed{\textit{analysis}}$ integral [1] (for $k \neq 0$)

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jk\omega_0 t} dt$$

to find the <u>first</u> ($k = 1$) Fourier series coefficient, a_1. Note: $\omega_0 = 2\pi / T_0$.

(d) If we add a constant value of one to $x(t)$, we obtain the signal $y(t) = 1 + x(t)$ with $y(t)$ given over one period by

$$y(t) = \begin{cases} 1 & 0 \le t \le 5 \\ 3 & 5 < t \le 10. \end{cases}$$

This signal can also be represented by a Fourier series,

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}.$$

Explain how b_0 and b_1 are related to a_0 and a_1. (Note: You should not have to evaluate any new integrals explicitly to answer this question.)

[1] The Fourier integral can be done over any period of the signal; in this case, the most convenient choice is from -5 to 0.