PROBLEM:

Complex exponentials obey the expected rules of algebra when doing operations such as integrals, derivatives, and time-shifts. Consider the complex signal $z(t) = Ze^{j20\pi t}$ where $Z = 2e^{j\pi/3}$.

(a) Show that the first derivative of $z(t)$ with respect to time can be represented as a new complex exponential $Qe^{j20\pi t}$, i.e., $\frac{d}{dt}z(t) = Qe^{j20\pi t}$. Determine the value for the complex amplitude Q.

(b) Plot both Z and Q in the complex plane. How much greater (or smaller) is the angle of Q than the angle of Z?

(c) Compare $\Re e\{\frac{d}{dt}z(t)\}$ to $\frac{d}{dt}\Re e\{z\}$ for the given signal $z(t)$. Do you think that this would be true for any complex exponential signal?

(d) Evaluate the definite integral of $z(t)$ over the range $-0.05 \le t \le 0.05$:

$$\int_{-0.05}^{0.05} z(t)dt = ?$$

Note that integrating a complex quantity follows the expected rules of algebra: you could integrate the real and imaginary parts separately, but you can also *use the integration formula for an exponential* directly on $z(t)$.

(e) Show that the time-shifted version of $z(t)$ can be represented as a new complex exponential $Pe^{j20\pi t}$, i.e., $z(t - t_d) = Pe^{j20\pi t}$. Determine the value for the complex amplitude P. Use your result to determine how t_d be should chosen so that $z(t - t_d) = z(t)$.